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Identification of fractional-derivative-model parameters of viscoelastic materials from measured FRFs

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Abstract

The dynamic properties of viscoelastic damping materials are highly frequency- and temperature-dependent. Numerical methods of structural and acoustic systems require the mathematical model for these dependencies. The fractional-derivative model on damping material has become a powerful solution that describes the frequency-dependent dynamic characteristics of damping materials. The model parameters on a damping material are very important information both for describing the responses of damped structures and in the design of damped structures. The authors proposed an efficient identification method of the material parameters using an optimization technique, showing its applicability through numerical studies in a previous work. In this study, the proposed procedure is applied to a damping material to identify the fractional-derivative-model parameters of viscoelastic materials. In the proposed method, frequency response functions (FRFs) are measured via a cantilever beam impact test. The FRFs on the points identical to those measured are calculated using an FE model with the equivalent stiffness approach. The differences between the measured and the calculated FRFs are minimized using a gradient-based optimization algorithm in order to estimate the true values of the parameters. The FRFs of a damped beam structure are measured in an environmental chamber at different temperatures and used as reference responses. A light impact hammer and a laser vibrometer are used to measure the reference responses. Both linear and nonlinear relationships between the logarithmically scaled shift factors and temperatures are examined during the identification of the material parameters. The applied results show that the proposed method accurately identifies the fractional-derivative-model parameters of a viscoelastic material.

1. Introduction

Currently, increasingly lightweight designs of mechanical products such as automobiles, ships, and electric appliances have become common in order to improve fuel economy and to reduce the costs of production. However, this tendency is frequently associated with vibration and noise problems. In these cases, viscoelastic damping materials have become widely used to suppress mechanical vibrations [1]. Placement of constrained or unconstrained viscoelastic damping layers on the surface of structures is a typical method to introduce damping in structures [2–4]. To predict the responses of the damped structures, one must know the material

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properties of the damping material. Therefore, knowledge of the material properties of a viscoelastic damping material is very important for describing the responses of a damped structure and in the design of the damped structure.

The dynamic properties of viscoelastic damping materials show strong frequency- and temperature-dependencies. Generally the storage modulus of damping material increases monotonically along frequency axis, and the loss factor shows a wide peak along the frequency axis. Operational temperature of viscoelastic damping material varies the dynamic properties by shifting the frequency characteristics on frequency axis. To describe the frequency- and temperature-dependent dynamics properties efficiently, computerized numerical analysis methods such as, for example, finite element method require a mathematical model on the dynamic properties. In recent years, the fractional-derivative model has been used to describe the dynamic characteristics of viscoelastic damping materials that are dependent upon the frequency and temperature [5,6]. The fractional-derivative model can easily and simply describe the real dynamic behavior of viscoelastic damping materials using only four parameters [7–9]. To apply the mathematical form to a damping material, the model parameters should be easily estimated through experiments because there are not only many kinds of damping material but also variation in polymeric composition. The fractional-derivative-model parameters can be identified from the measured elastic moduli and loss factors using a curve-fitting process [10,11]. However, to obtain the elastic moduli and loss factors of a viscoelastic damping material using the conventional method, many tests are typically required at different temperatures, as the elastic moduli and loss factors are detected only at the resonant frequencies of frequency response functions (FRFs). After collecting a large amount of data at different frequencies and temperatures in the conventional method, experimental data are compared with theoretical values from the mathematical model. The difference between the experimental data and the theoretical values must be minimized by changing the model parameters in order to estimate the parameters of the fractional-derivative model [12–14]. A curve-fitting process then gives the fractional-derivative model of a viscoelastic material, which can describe frequency-dependent dynamic characteristics of the damping material over a concerned reduced frequency range. Here, it should be noted that an efficient updating scheme of the parameters is needed in the curve-fitting process for the rapid identification of the material properties. However, the conventional trial-and-error approach is especially time-consuming in terms of the data collection step, as well as the curve-fitting process. The dynamic characteristics of viscoelastic damping materials can vary significantly depending on the combination of polymers, and many kinds of damping materials are produced for special purposes. In this situation the conventional method is not satisfactory to rapidly provide the material parameter information of a damping material. Consequently, an efficient identification method to estimate the fractional-derivative-model parameters of damping material is needed.

Several studies have attempted to identify the material properties of viscoelastic damping materials. Lekszycki et al. [15] investigated the identification problem of the constitutive parameters of a viscoelastic material using a one-dimensional Voigt model and the optimality conditions in a constrained beam. Deng et al. [16] proposed a system identification procedure based on direct nonlinear optimization and sub-optimal methods to estimate the viscoelastic parameters of polyurethane foam modeled by a fractional-derivative model. Lee and Hwang [17] proposed a structural joint identification method in a real structure in which the differences between calculated FRFs and measured FRFs are minimized using a gradient-based optimization method. Kim and Lee [18] also proposed an efficient identification method of the material parameters using an optimization technique and showed its applicability through numerical examples.

In this paper, the earlier work [18] is extended for a real viscoelastic damping material to identify the fractional-derivative-model parameters of a damping material. An identification procedure of the fractional-derivative-model parameters is proposed using a finite element model of an unconstrained beam and a gradient-based numerical search algorithm. In the proposed method, the measured FRFs of a damped beam are used not at the resonant frequencies but at all frequencies so that the number of required experiments can be reduced considerably. In addition, analytic sensitivity formulae are derived and used in the curve-fitting process in order to speed up the identification process. For robustness and efficiency, the identification procedure is divided into two steps: the first is a resonant frequency alignment step and the second is minimization of the difference between measured FRFs data and simulated FRFs ones. The proposed method is applied to a viscoelastic damping material to demonstrate its correctness and efficiency.

2. Description of viscoelastic vibration damping

2.1. Fractional-derivative model

Viscoelastic behavior occurs in a wide range of materials and can be characterized by liquid-like elastic behavior. Materials that experience viscoelastic behavior include acrylics, rubber and adhesives. For these materials, a linear elastic constitutive relationship using Hook's law is not an accurate representation of the dynamic characteristics when attempting to explain such factors as creep and stress relaxation over short time scales. Instead, the complex modulus concept is extensively used to describe the dynamic characteristics of viscoelastic materials. The complex modulus is employed using the approach of the constitutive relationship of Hook's law in the frequency domain as follows:

$$\overline{\sigma} = E^{*}\overline{\varepsilon} = E(1 + \mathrm{i}\eta)\overline{\varepsilon}, \tag{1}$$

where E^{*} is the complex modulus, and $\overline{\sigma}$ and $\overline{\varepsilon}$ are the Fourier transforms of stress and strain, respectively.

The complex modulus of a viscoelastic material depends on temperature and frequency. To include the effects of temperature into the dynamic behavior of viscoelastic materials, the temperature–frequency superposition principle [12] can be used. This converts the temperature effects into those of frequency. From the temperature–frequency equivalence hypothesis, the complex modulus values at any frequency f_1 and any reference temperature T_1 are identical to those at any other frequency f_2 at a different temperature T_2 if the shift factor $\alpha(T_2)$ is determined as follows:

$$E^{*}(f_1, T_1) = E^{*}(f_2\alpha(T_2), T_2), \tag{2}$$

where $f\alpha(T)$ refers to the reduced frequency. Therefore, preparing a master curve for the complex modulus of a viscoelastic material against frequency at a reference temperature T_0 in absolute degrees, it is easy to predict the complex modulus at any other temperature using the shift factor.

For many viscoelastic materials the shift factor in the logarithmic scale is inversely proportional to temperature over a wide temperature range. Accordingly, the Arrhenius Eq. (3) can be used to approximate the relationship as follows:

$$\log(\alpha(T)) = d_1\left(\frac{1}{T} - \frac{1}{T_0}\right). \tag{3}$$

Here, d_1 is a material constant. However, it is also known that a linear Arrhenius relationship can deviate from experimental data at higher and lower temperatures. In this case, a nonlinear relationship that can be used is the William–Landel–Ferry (WLF) Eq. (4):

$$\log(\alpha(T)) = d_1\frac{(T - T_0)}{(b_1 + T - T_0)}, \tag{4}$$

where b_1 is a material constant.

To mathematically describe the dynamic characteristics of the complex modulus of the viscoelastic materials, constitutive equations that relate stresses and strains should be known. The fractional-derivative model represents the damping elements as a time derivative of an order smaller than unity. The constitutive equation of the four-parameter fractional-derivative model can be written as follows:

$$\sigma(t) + c_1 D^{\beta}\sigma(t) = a_0\varepsilon(t) + a_1 D^{\beta}\varepsilon(t), \tag{5}$$

where $0<\beta<1$, and a_0, a_1, c_1 and β are material parameters. D^{β} indicates the fractional-order derivative, as in

$$D^{\beta}\sigma(t) = \frac{1}{\Gamma(1-\beta)}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^t \frac{\sigma(\tau)}{(t-\tau)^{\beta}}\,\mathrm{d}\tau, \tag{6}$$

where $\Gamma(x)$ is the Gamma function. It should be pointed out that the four-parameter fractional-derivative model must satisfy the thermodynamic requirements of nonnegative internal work and the nonnegative energy dissipation rate. The fractional-derivative model satisfies the thermodynamic constraints if the four parameters are positive and $a_1/c_1 \geqslant a_0$ [19]. The complex modulus of the fractional-derivative model can be obtained by the Fourier transform of Eq. (5):

$$\overline{\sigma} = \frac{a_0 + a_1(\mathrm{i}\omega)^\beta}{1 + c_1(\mathrm{i}\omega)^\beta}\overline{\varepsilon} = E^*\overline{\varepsilon}. \tag{7}$$

Here, ω is the angular velocity. Introducing the reduced frequency in order to consider temperature effects into Eq. (7), the complex modulus of viscoelastic materials are finally expressed as

$$E^* = E(1 + \mathrm{i}\eta) = \frac{a_0 + a_1[\mathrm{if}\ \alpha(T)]^\beta}{1 + c_1[\mathrm{if}\ \alpha(T)]^\beta}. \tag{8}$$

It is well known that the fractional-derivative model can sufficiently represent the real behavior of viscoelastic materials over a wide frequency range [7]. Therefore, when identifying the material parameters of viscoelastic materials, the fractional-derivative model can describe the dynamic characteristics of the viscoelastic materials over frequency and temperature variations.

To estimate the fractional-derivative-model parameters of an unknown material with conventional methods, many initial tests should be repeated until a sufficient number of complex moduli are acquired at different frequencies and temperatures using, for example, the Oberst beam test, as shown in Fig. 1. Next, from the acquired data, the coefficients of the fractional-derivative model are determined using a statistical data analysis technique that minimizes the mean-square error between the theoretical values and the tabulated values. However, the statistical data analysis process and the data collection step are time-consuming, as trial-and-error steps are involved; specifically, the reference temperature is assumed and the mean-square error is minimized between the theoretical values and the experimental data. The trial-and-error step is repeated in turn until the global minimum value of the least-square error is obtained.

2.2. Finite element analysis of a damped beam

Various methods are available for the response analysis of the unconstrained-layer-damping beam shown in Fig. 2. Among those, the equivalent rigidity method of Ross, Kerwin, and Ungar (RKU) is a popular method not only for its simple form but also due to its accuracy. For the example of the unconstrained damping layer beam, the storage modulus and the loss factor of the viscoelastic damping layer are E_2 and η_2, respectively. The storage modulus, the loss factor, and the second area moment of the base beam are E_1, η_1, and I_1, respectively. The equivalent complex flexural rigidity, E^*I, of the unconstrained beams using the RKU equation is written as [12]

$$\frac{E^*I}{E_1^*I_1} = 1 + e^*h^3 + 3(1+h)^3\frac{e^*h}{1+e^*h}, \tag{9}$$

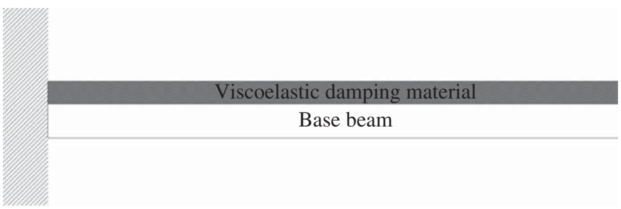

Fig. 1. Oberst beam test configuration.

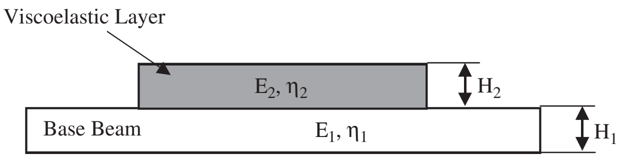

Fig. 2. Unconstrained damping layer beam.

where $h = H_2/H_1$, $e^* = E_2^*/E_1^*$, and the superscript (*) refers to complex quantity. From the RKU equation, the equivalent storage modulus of the unconstrained-layer-damping beam is the real part of Eq. (9), and the equivalent loss factor can also be obtained from the imaginary part of Eq. (9). Thus, the unconstrained-damping-layer beam can be analyzed using a finite beam element formulation with the equivalent flexural rigidity. Introducing a finite beam element with flexural displacement and rotations at the nodes with cubic shape functions and ignoring the shear deformation as the unconstrained-damping-layer beam undergoes primarily extensional bending, it is possible to obtain the equations of motion, as follows:

$$[\mathbf{M}]\{\ddot{\mathbf{x}}\} + [\mathbf{K}]\{\mathbf{x}\} = \{\mathbf{f}\}. \tag{10}$$

Here, [**M**] and [**K**] are the global mass and stiffness matrices, respectively, and {**x**} and {**f**} are the displacement and force vectors, respectively. Here, the matrix [**K**] is a complex-valued matrix because the equivalent stiffness of the viscoelastic damping layer beam becomes a complex quantity. In addition, the matrix [**K**] satisfies the following relationship:

$$[\mathbf{K}] = \mathrm{Re}[\mathbf{K}] + \mathrm{i} \cdot \mathrm{Im}[\mathbf{K}] = \mathrm{Re}[\mathbf{K}](1 + \mathrm{i}\eta), \tag{11}$$

in which Re and Im refer to real and imaginary parts, respectively. Assuming the harmonic motion of the system, the corresponding eigenvalue problem can be defined as

$$\mathrm{Re}[\mathbf{K}]\{\mathbf{y}\} = \varsigma[\mathbf{M}]\{\mathbf{y}\}, \tag{12}$$

where the vector {**y**} is the eigenvector and $\varsigma(= \omega^2 = (2\pi f)^2)$ is the eigenvalue. The eigenvalue problem of Eq. (12) is a frequency-dependent equation because the stiffness matrix is a function of the frequency due to the viscoelastic damping layer. To solve the frequency-dependent eigenvalue problem, an iteration scheme is necessary. In this study, a simple re-substitution method is used. The first step of the method is the assumption of the eigen-frequency, f_0, for a given temperature. It is then possible to evaluate the complex modulus of the viscoelastic material from Eq. (8); consequently, the equivalent stiffness of the elements can be calculated using Eq. (9). Next, solving the linear eigenvalue problem of the assembled equation, it is possible to obtain a new frequency and repeat iterations by updating the frequency. If the difference in the frequencies during the iterations converges to zero, the iteration stops.

The response of the unconstrained-layer-damping beam is calculated using the modal superposition method. The modal superposition principle gives an expression of the harmonic responses in vibration problems. The displacement of the damped structure can then be written as

$$\{x\} = \sum_{k=1}^{m} a_k \{y_k\}, \tag{13}$$

where m and y_k are the number of modes and the k-th eigenvector, respectively, and a_k is the k-th modal coordinate, as follows:

$$a_k = \frac{\{y_k\}^{\mathrm{T}}\{F\}}{(\zeta(1 + \mathrm{i}\eta_k) - \omega^2)}. \tag{14}$$

Here, $\{F\}$ and η_k are the harmonic nodal force vector and the loss factor of the k-th mode, respectively. The loss factor of a structure for a vibration mode is defined as

$$\eta_k = \sum_{j=1}^{p} \eta^j U_{ej} \Bigg/ \sum_{j=1}^{p} U_{ej} = \sum_{j=1}^{p} \eta^j U_{ej} \Bigg/ U, \tag{15}$$

where p is the number of finite elements and η^j is the loss factor of the j-th element. Additionally, U_{ej} is the strain energy of the j-th finite element and U represents the total strain energy.

3. Identification of the viscoelastic material parameters

3.1. A proposed identification method

To develop an efficient estimation method of the fractional-derivative-model parameters, in this paper it is assumed that if a numerical model reproduces measured responses, the parameters of the material used in the simulation model are then identical to the true values of the material properties. Thus, by minimizing the response difference between the measured and simulated FRFs using a numerical search algorithm, it is possible to identify the material properties. The basic idea is adopted from a previous study [18]. It is common in many inverse problems that the least-square error between two responses is selected as a measure of the response difference. Therefore, the summation of the square of the difference between the simulated and measured FRFs over a concerned frequency range can be defined as an identification index, as follows:

$$g(b) = \sum_{i=1}^{N} \int (x_s^i - x_m^i)^2 \, \mathrm{d}f, \tag{16}$$

where x, N, and f are the frequency responses, the number of reference responses, and the frequency, respectively. The subscripts s and m denote the simulated and measured FRFs, respectively. Generally, gradient-based mathematical programming techniques are used to minimize the identification index function as gradient-based methods are the most efficient, although they may give a local minimum. The convex region of the identification index function should be as wide as possible in order for the identification procedure to consistently yield true values regardless of initial values. In order to widen the convergent region and make the method of the identification process efficient, the identification procedure is divided into two steps with proper identification index functions. The first step is the resonant-frequency alignment step, as illustrated in Fig. 3(a), and the second step is the amplitude-alignment step shown in Fig. 3(b). In the first step, only the square error of the resonant frequencies is minimized so that the simulated model can roughly approximate the experimental frequency responses before a fine adjustment of the frequency responses is achieved. Therefore, the identification index defined in Eq. (16) is split into two functions, as follows:

Step 1:

$$g_1(b) = \sum_{i=1}^{N} \sum_{k=1}^{M} (\lambda_{k,m}^i - \lambda_{k,s}^i)^2. \tag{17}$$

Step 2:

$$g_2(b) = \sum_{i=1}^{N} \int [20 \log(x_m^i) - 20 \log(x_s^i)]^2 \, \mathrm{d}f. \tag{18}$$

Here, λ and M are the resonant frequency and the number of resonant peaks within the relevant frequency range, respectively. Thus, minimizing the first identification index function with respect to the parameters of the fractional-derivative model, the response differences will be very small. The second step is the minimization of the magnitude difference between the measured and simulated FRF, which can be started from values close to the true values. Accordingly, the two-step identification procedure can significantly reduce the possibility of falling into a local minimum. Furthermore, as shown in the preliminary numerical study by the authors [18], the two-step procedure can greatly reduce the computational cost of the identification procedure; in the first step, only the resonant frequencies rather than every frequency response over a concerned frequency range are necessary. Fig. 4 shows the contour surfaces of the identification index functions for a typical damped beam problem [18] according to the parameters normalized to the true values. As shown in Fig. 4, the first identification index function is sufficiently smooth over the wide region, and the

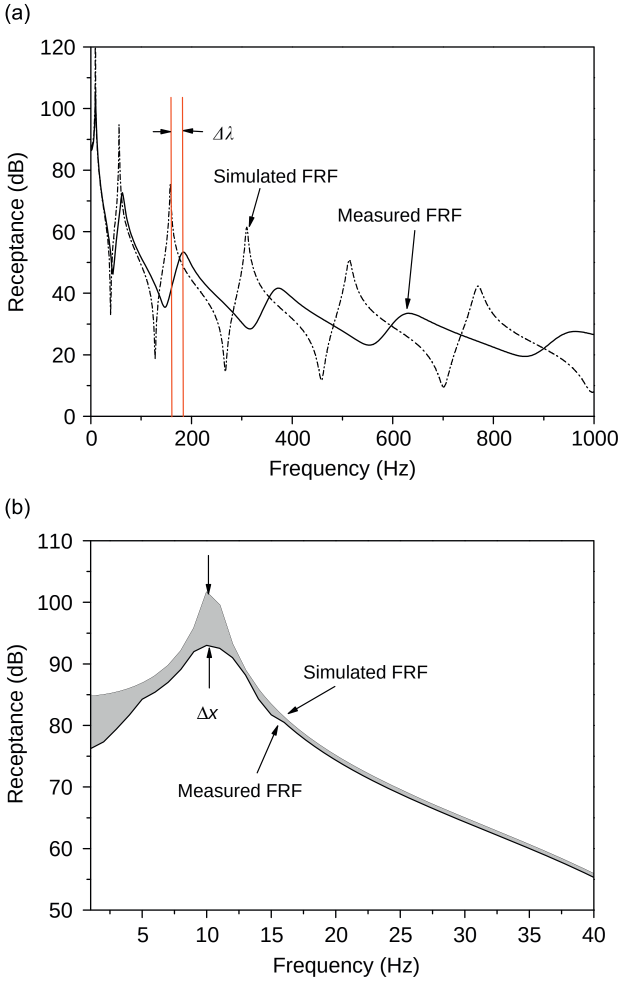

Fig. 3. Two-step identification method: (a) step 1: resonant frequency step and (b) step 2: amplitude alignment step.

second identification index function is very smooth near the true values. The iterative procedure of the identification is summarized in Fig. 5.

3.2. Parameters sensitivity analysis

The fractional-derivative-model parameters can be identified using a gradient-based numerical search method. In order to identify the material parameters using gradient-based algorithms, a parameter sensitivity analysis for the response is necessary. Parameter sensitivity is the gradient of a function with respect to a parameter. The computational cost of the iterative identification procedure is heavily dependent upon the efficiency of the parameter sensitivity analysis. In this study, an analytical formulation of the sensitivity information is used for the frequency responses and eigenvalues. As a simple alternative method, the finite difference method can be used to calculate the design sensitivity information. However, the finite difference method is very expensive and loses accuracy near the minimum point, which results in a slow convergence rate of the numerical search algorithm. In this study a direct differentiation method [20,21] is applied to the discrete system equations to obtain an analytic design sensitivity formula with respect to the material parameters for an FRF.

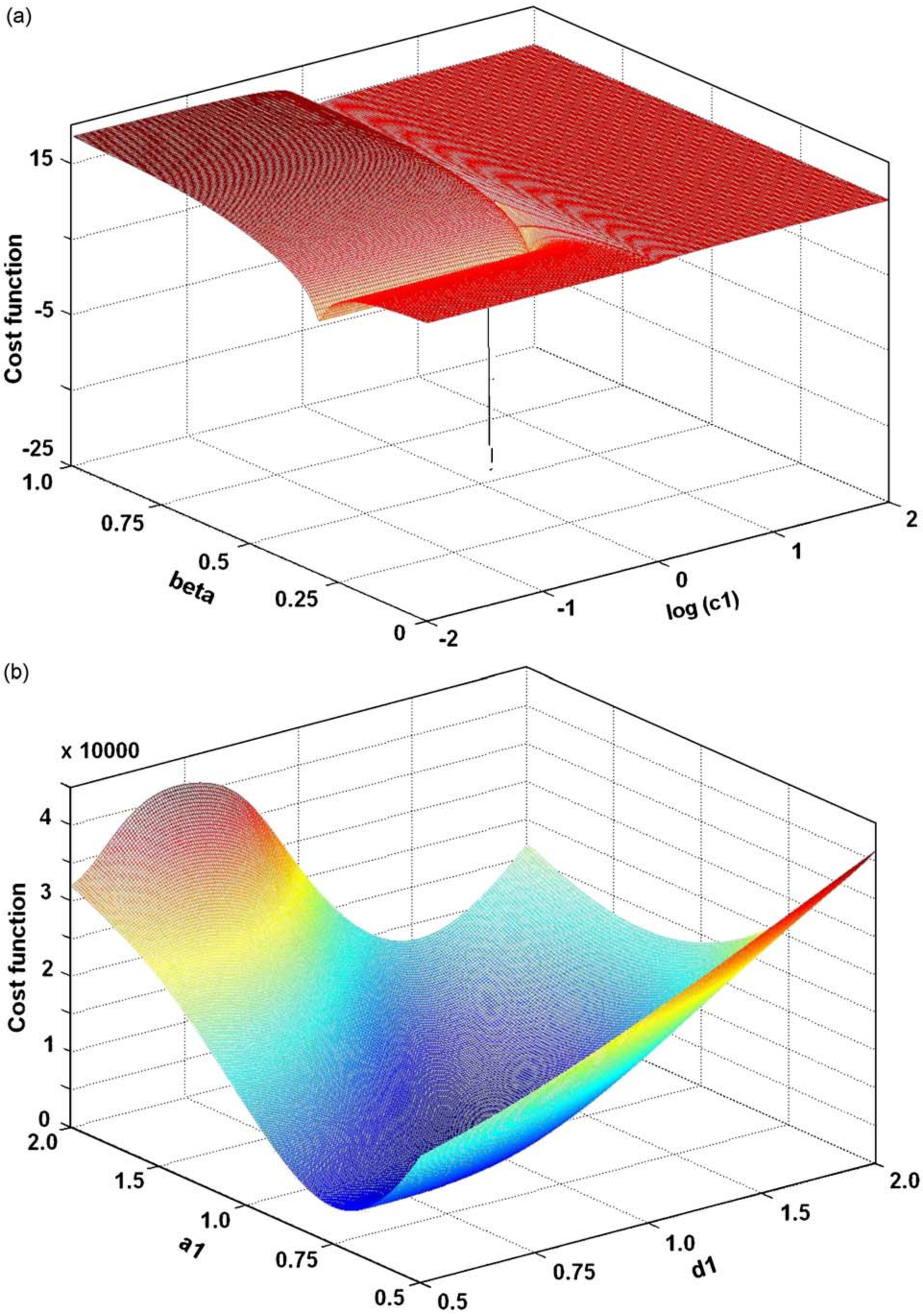

Fig. 4. Contour surfaces of the identification index: (a) first step: $\log(c_1)$ vs. β and (b) second step: a_1 vs. d_1.

The response of the unconstrained-layer-damping beam is expressed as Eq. (13). The parameter sensitivity information is obtained by differentiating the response expression with respect to the fractional-derivative-model parameters, as follows:

$$\frac{\mathrm{d}x}{\mathrm{d}b} = \sum_{k=1}^{p}\left(\frac{\mathrm{d}a_k}{\mathrm{d}b}\{y_k\} + a_k\left\{\frac{\mathrm{d}y_k}{\mathrm{d}b}\right\}\right), \tag{19}$$

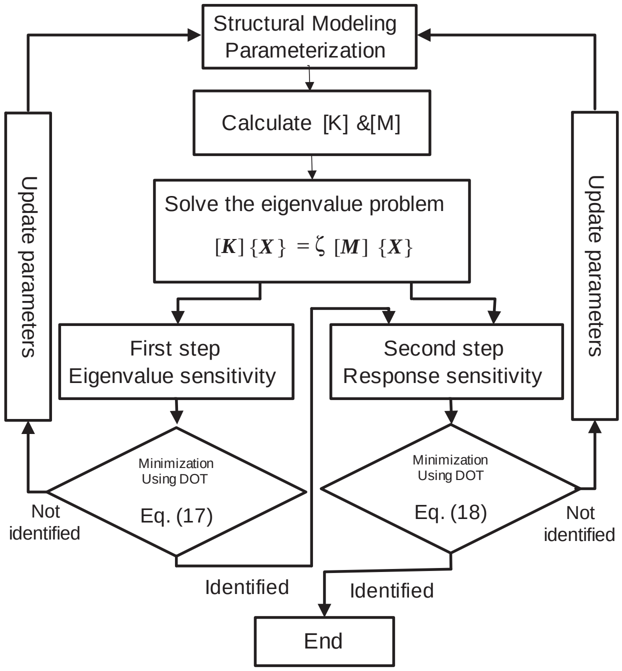

Fig. 5. Flowchart of the two-step identification procedure.

where

$$\frac{\mathrm{d}a_k}{\mathrm{d}b} = \left\{\frac{\mathrm{d}y_k}{\mathrm{d}b}\right\}^{\mathrm{T}} \cdot \{F\} \cdot (\zeta_k(1+\mathrm{i}\eta_k) - \omega^2)^{-1} - \frac{\{y_k\}^{\mathrm{T}}\{F\}\left(\frac{\mathrm{d}\zeta}{\mathrm{d}b}(1+\mathrm{i}\eta) + \mathrm{i}\zeta_k\frac{\mathrm{d}\eta_k}{\mathrm{d}b}\right)}{(\zeta_k(1+\mathrm{i}\eta_k)-\omega^2)^2}, \tag{20}$$

where b denotes the fractional-derivative-model parameters. Investigating the sensitivity expression of Eq. (20), the parameters sensitivity of the response can be calculated from the sensitivity of the eigenvalue, eigenvector, loss factor, and derivative expression of the complex modulus represented by the fractional-derivative model.

The eigenvalue sensitivity formula for the k-th eigenvalue, $\mathrm{d}\zeta_k/\mathrm{d}b$, in discrete form is obtained by differentiating the eigenvalue problem equation for the k-th eigenvalue and eigenvector. Premultiplying the k-th eigenvector with the resulting equation and applying the orthogonal condition, the eigenvalue sensitivity formula can be obtained by the following equation:

$$\dot{\varsigma}^i = \{y^i\}^{\mathrm{T}}\left(\left[\frac{\partial K}{\partial b}\right] - \varsigma^i\left[\frac{\partial M}{\partial b}\right]\right)\{y^i\}. \tag{21}$$

The details of the parameters sensitivity formula can be found in Ref. [18].

4. Application to a damping material

To verify the identification method that was proposed in the previous section, a viscoelastic damping material, 3M-467 adhesive, is selected, and its fractional-derivative-model parameters will be identified in this section. To obtain the reference responses of the identification method at several temperatures, impact hammer tests in a constant-temperature chamber were carried out. Figs. 6 and 7 show a schematic diagram of the test set-up and the clamped beam structure, respectively. The beam-clamping structure was composed of a steel jig fixed on a test bench and an aluminum beam. The length, width, and thickness of the beam were 200.0, 20.0, and 4.0 mm, respectively. The aluminum beam was clamped by two plates fastened by six steel bolts with a constant torque. A Polytec Laser Doppler Velocimetry (LDV) and Cada-X software [22] were used to

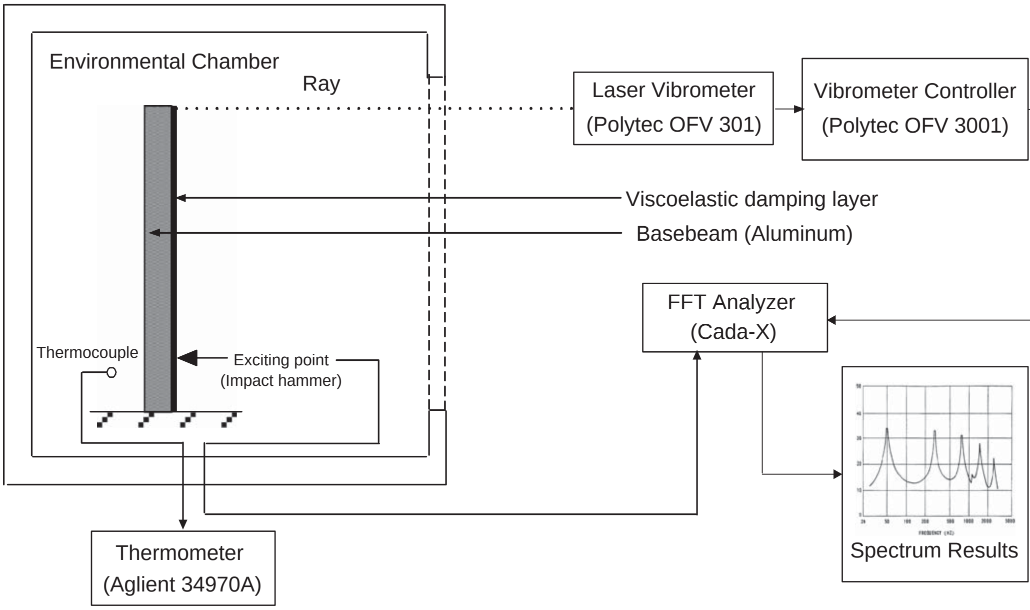

Fig. 6. Schematic diagram of the experimental set-up.

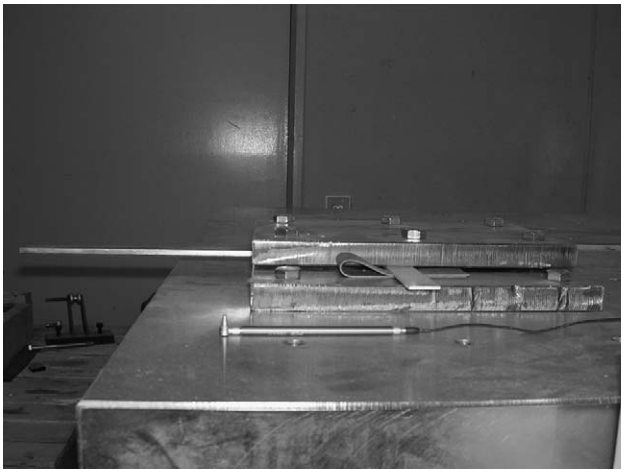

Fig. 7. Experimental set-up for the clamped-free beam.

acquire the response signals. Only one directional component perpendicular to the beam was excited in the experiment by impacting the beam on the free end point of the center line and was also measured by the LDV. The frequency band was 3000 Hz with a frequency resolution of 1 Hz.

A finite element model for the bare beam was developed and validated. The bare beam was modeled by 40 linear finite elements. The exact material properties of the bare beam should be known prior to applying the proposed method to a damping material. The Young's modulus value of aluminum is well-known; however, it has small variation according to samples. To estimate the exact material properties of the bare beam, the calculated FRF of the bare beam was correlated with the measured response of the bare beam by slightly changing the Young's modulus and structural damping values from the typical values of aluminum. Fig. 8 shows the experimental and the calculated FRFs for the bare beam. In Fig. 8, one can see very good agreement between the two results. The estimated material properties of the bare beam were used for the FE model as the

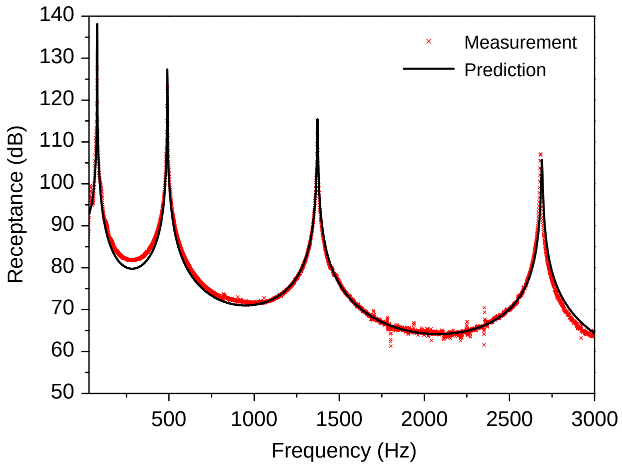

Fig. 8. Calculated FRF compared with measured one for the bare beam.

Table 1
The material properties of aluminum beam.

Young's modulus	6.91 GPa
Density	2780 kg/m^3
Poisson's ratio	0.31
Damping coefficient	0.001

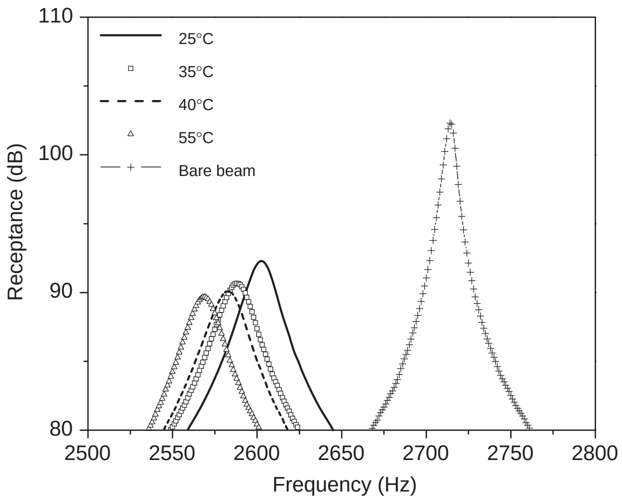

Fig. 9. Measured FRFs at different temperatures for the 3M-467 coated beam.

exact value material properties of the bare beam. The material properties used in the FE model are listed in Table 1.

A viscoelastic material, 3M-467 adhesive, was bonded onto the bare beam at a thickness of 1.2 mm. The length and width of the viscoelastic damping layer were identical to those of the base beam. The fractional-derivative-model parameters of the viscoelastic material were then identified using the proposed method. The reference FRFs were measured at the four different temperatures of 25, 35, 40, and 55 °C. To measure the FRFs at a temperature, the temperature of the environment chamber was kept at the given temperature at least 2 h. The measured responses were averaged seven times for each temperature. Fig. 9 shows the fourth peak of the measured FRFs. It can be shown in Fig. 9 that a hotter viscoelastic material results in higher damping and lower stiffness. The damped beam was also modeled using 40 finite elements with the equivalent stiffness. Subsequently, the identification index function was defined and minimized in order to identify the material parameters. The lower and upper limits of the frequency band for the index function were 30 and 3000 Hz, respectively. To solve the minimization problem, a commercial program, DOT Ver. 5.4 [23], was employed with the analytical sensitivity information. To numerically validate the proposed parameters sensitivity analysis procedure, the sensitivity results of the damped beam at the initial configuration were compared with those of the finite difference method. The initial parameters of the fractional-derivative model are as follows: $a_0 = 3.0$, $a_1 = 5.0$, $c_1 = 0.1$, $b_1 = 380$, $d_1 = 12$, $\beta = 0.5$, and $T_0 = 30\,^\circ\mathrm{C}$. The sensitivities of fractional-derivative-model parameters for the FRFs at the free end of the beam were then calculated. The calculated fractional-derivative-model parameter sensitivities were compared with those of the forward finite difference method, as shown below:

$$\frac{\partial x}{\partial b} \cong \frac{\Delta x}{\Delta b} = \frac{x(b+\Delta b) - x(b)}{\Delta b}. \tag{22}$$

Here, Δb is the amount of perturbation in the fractional-derivative-model parameters. For the finite difference method, perturbation by 0.01 percent of the parameter was used. Fig. 10 shows that the two results are in good agreement, which proves that the presented parameters sensitivity formulation and the numerical implementations were carried out correctly.

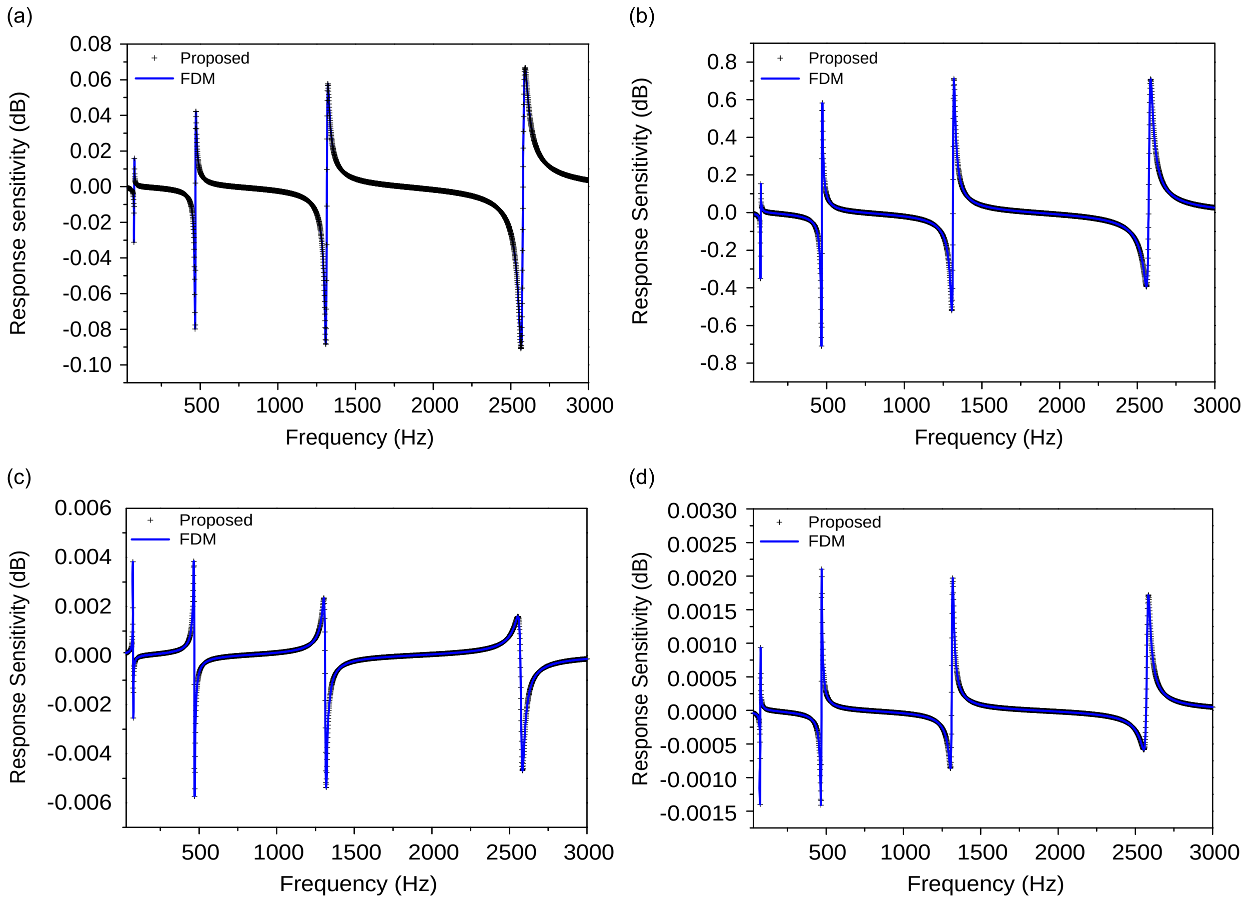

Fig. 10. Response sensitivities with respect to the material parameters compared with those of FDM using WLF shift factor equation: (a) a_1, (b) β, (c) d_1, and (d) T_0.

By identifying the fractional-derivative-model parameter using the proposed identification procedure, it is possible to select the linear Arrhenius or the nonlinear WLF relationships to define the relationship between the shift factor and temperature. If the linear relationship is used, it was shown in an earlier study [18] that two reference FRFs are sufficient when identifying the parameters. However, a number of FRFs over the relevant temperature range must be given for a nonlinear WLF relationship. First, the parameters were identified with the four reference FRFs measured at the different temperatures using the linear Arrhenius relationship. Table 2 shows the identified parameters of the fractional-derivative model with the Arrhenius relationship. Comparing the simulated responses calculated with the identified parameters with the reference FRFs, the regenerated FRFs at 25, 35, and 40 °C were in very good agreement with the reference FRFs. However, as shown in Fig. 11, the reference FRF and the calculated one at 55 °C showed a slight difference in the location of the fourth resonant frequency, which indicates that the linear relationship of the shift factor does not

Table 2
Identified material parameters for 3M-467 adhesive using the Arrhenius shift factor relationship.

Parameters	Initial value	First step	Second step
a_0	1.00E−02	0.44414E+00	0.34132E+00
a_1	1.00E−02	0.87051E+00	0.49361E+00
c_1	1.00E−02	0.15393E−03	0.10205E−03
d_1	1.00E+00	0.48571E+04	0.52783E+04
β	1.00E+00	0.56116E+00	0.52901E+00
T_0	1.00E+01	0.26107E+02	0.23703E+02

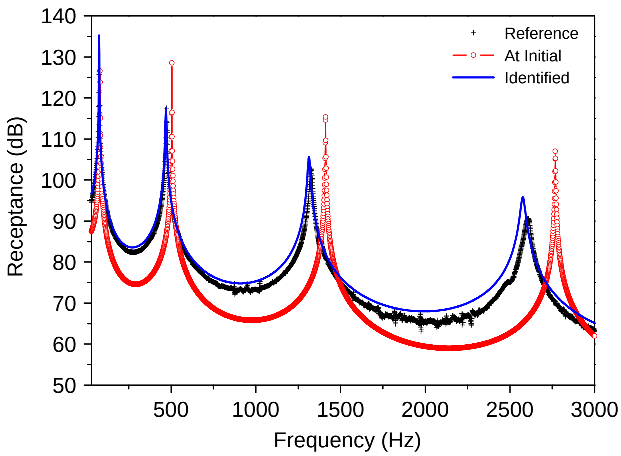

Fig. 11. Regenerated FRF at 55 °C using the Arrhenius shift factor relationship compared with that of measured one.

Table 3
Identified material parameters for 3M-467 adhesive using the WLF shift factor relationship.

Parameters	Initial value	First step	Second step
a_0	1.00E−02	0.41589E+00	0.34065E+00
a_1	1.00E−02	0.86751E+00	0.49161E+00
c_1	1.00E−02	0.14993E−03	0.10212E−03
b_1	1.00E+00	0.15571E+02	0.19489E+02
d_1	1.00E+02	0.35621E+03	0.37681E+03
β	1.00E+00	0.56886E+00	0.52618E+00
T_0	1.00E+01	0.25707E+02	0.23527E+02

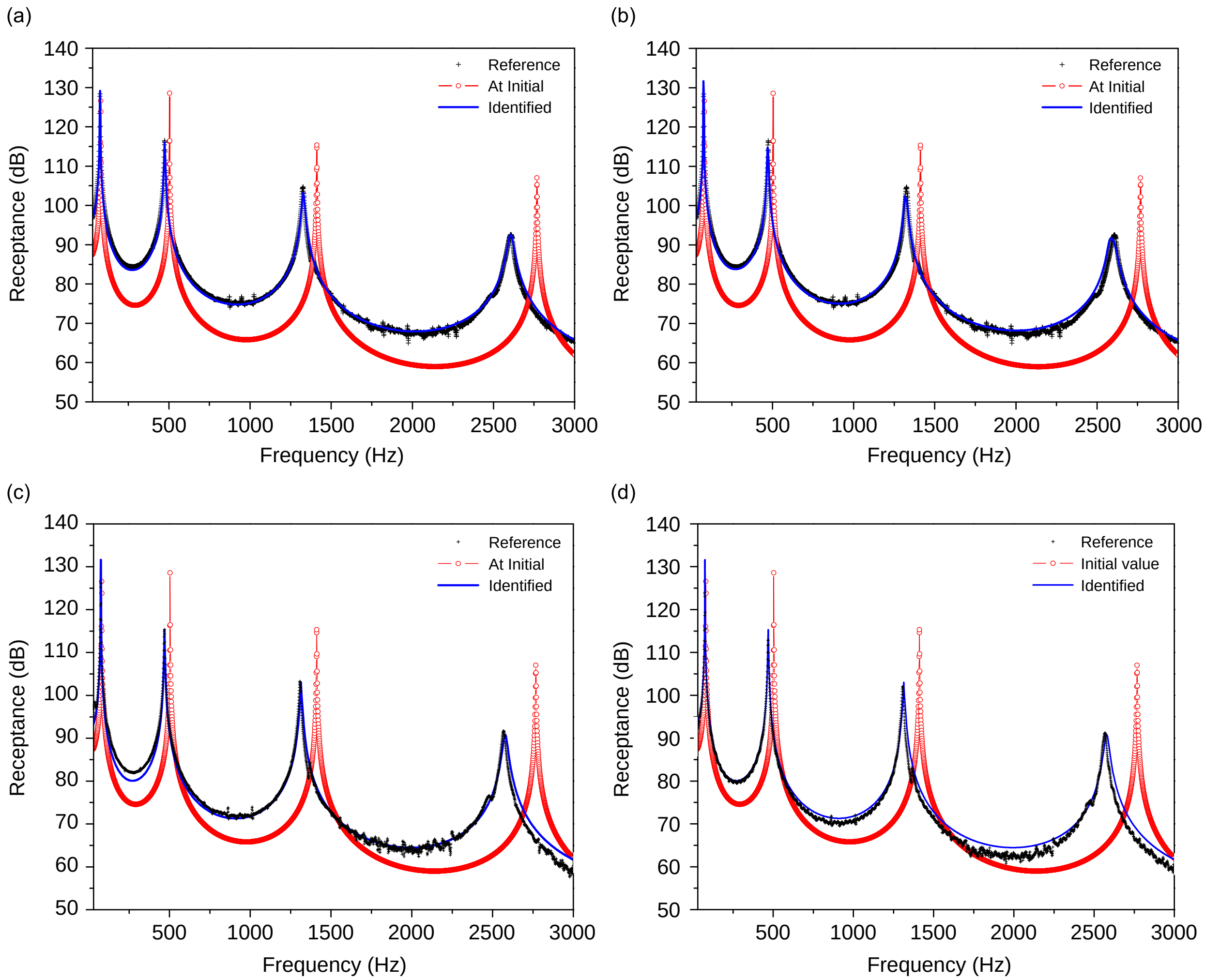

Fig. 12. Regenerated FRFs using the WLF relationship compared with the reference FRFs: (a) 25 °C, (b) 35 °C, (c) 40 °C, and (d) 55 °C.

properly describe the temperature effects of the material around 55 °C. Thus, the identification process was repeated once more under the same conditions apart from the use of the nonlinear WLF relationship. Table 3 shows the identified parameters of the fractional-derivative model with the WLF relationship. Fig. 12 also shows the regenerated FRFs with the identified parameters compared to the reference FRFs at 25, 35, 40, and 55 °C. One can see very good agreement between the regenerated FRFs and the reference FRFs.

In the second step of the identification procedure, only the magnitude information of the FRFs was used (the phase information was not used) to identify the parameters although the response is a complex quantity. This is because, in a previous numerical study on a structural joint identification problem [24], the second author tried to minimize the differences of the real and imaginary components of FRFs simultaneously, but the identified results worse than the magnitude-only approach. Therefore, the authors adopted the magnitude-only approach in this study. The proposed index function in this study has more advanced features than the previous one; that is, the two-step approach. In the first step, all eigen-frequencies are arranged in correct sequential order. This correct sequence of the eigen-frequencies sets the phase values close to those of the reference FRFs. In the second step, the magnitude differences are minimized. To minimize the magnitude differences, however, the authors use the complex-quantity sensitivity formula which has gradient values for the real and imaginary parts of the FRFs. The movement of the real and imaginary parts cannot be

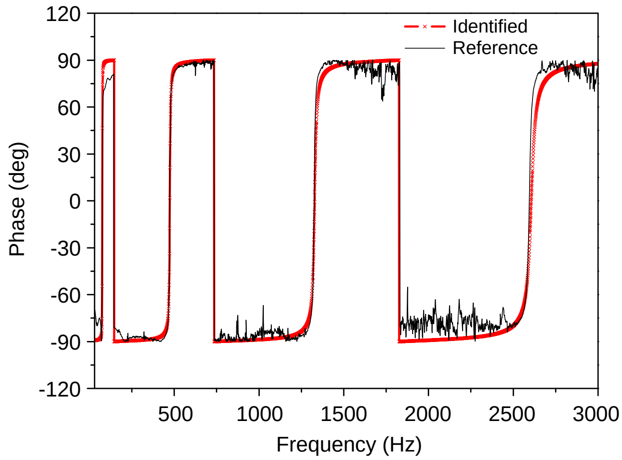

Fig. 13. Phase comparison of the regenerated FRF using the WLF relationship compared with the reference FRF ($T = 25$°C).

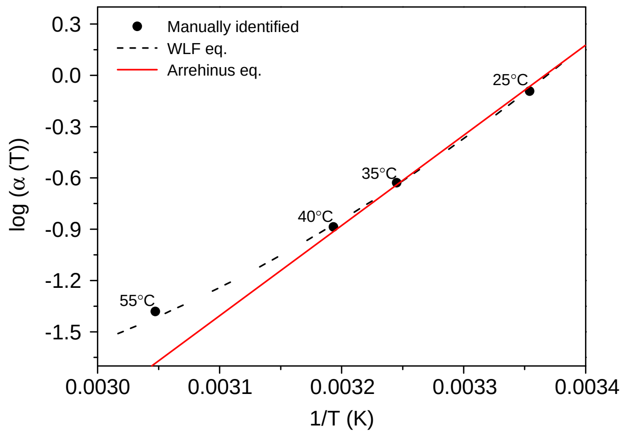

Fig. 14. The shift factor relationships compared with the manually identified values.

independent, but those are restricted by the amount of damping near resonant frequencies. The fractional-derivative-model parameters are determined so as to give the closest FRFs to the reference FRFs in an overall sense over a specified frequency range. Therefore, in most cases, under this approach the minimization of the magnitude differences resulted in good coincidence of phase values between the FRFs. Fig. 13 shows a phase comparison after the identification. One can see good agreement of the phase values.

As shown in Fig. 12, the nonlinear WLF relationship describes the temperature effect very well. It is possible to determine the shift factor value for each temperature manually by minimizing the response difference between the reference FRF and calculated value. These results are plotted with the identified Arrhenius and WLF relationships in Fig. 14. In this figure, the linear Arrhenius relationship begins to deviate from the genuine value above 40 °C.

Essentially, the fractional-derivative-model parameter values identified with the WLF relationship are very close to the listed material parameters of Ref. [12]. In Fig. 15, the material properties of the 3M-467 adhesive identified by the proposed method are compared with those of Ref. [12]. In Fig. 15, it was assumed that the Poisson's ratio of the adhesive is constant with respect to the frequency in order to convert the results of the present study. The Poisson's ratio used was 0.47. In Fig. 15, two results are shown to be in good agreement over the wide reduced frequency range.

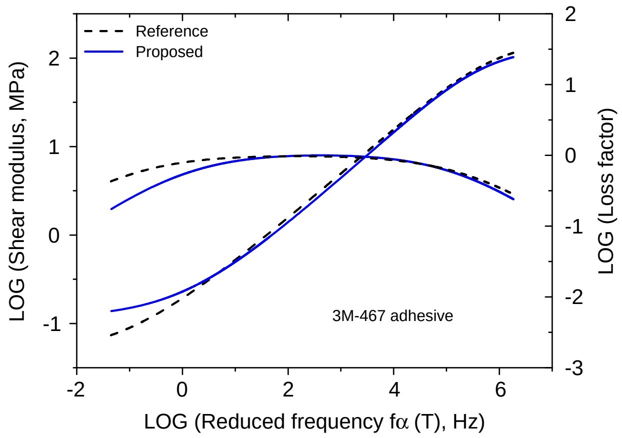

Fig. 15. Identified material properties of 3M-467 adhesive compared with Ref. [12].

5. Conclusions

To design and analyze damped structures, material properties, such as the elastic modulus and loss factor of the damping materials, are essential information. Viscoelastic materials show highly frequency- and temperature-dependent behavior. The four-parameter-fractional-derivative model can serve as a solution in many areas in order to represent the complex modulus and the loss factor of damping materials with respect to frequency and temperature. In this paper, an estimation method to obtain efficiently unknown fractional-derivative-model parameters of a real viscoelastic material is proposed. In the proposed method, the reference FRFs are measured using a cantilever beam impact hammer test at different temperatures. The FRFs of the points that are identical to the measured values are calculated using an FE model with the equivalent stiffness approach. The material properties are then identified using a numerical search algorithm by minimizing the response differences between the measured and simulated FRFs. The proposed method can reduce the number of required measurements for identification of the material properties. Moreover, data preparation using the proposed method is simplified, as the method uses raw FRFs directly as reference responses. In addition, the curve-fitting process is very fast because an efficient two-step approach is introduced and the analytic sensitivity formulae are used during the parameter-updating scheme.

The proposed method was applied to a damping material to estimate the fractional-derivative-model parameters of the damping material. Only four FRFs of a damped beam structure were measured at different temperatures for the identification of frequency- and temperature-dependent material properties. Both linear and nonlinear relationships between the logarithmically scaled shift factors and temperatures were examined in the identification of the material parameters. The estimation results show that the proposed method can accurately identify the fractional-derivative-model parameters for actual damping materials with minimal data preparation.

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